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# APPLICATION OF EFFICIENT METHODS FOR INSPECTION SENSITIVITY (PREPRINT)

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**14. ABSTRACT**
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# Application of Efficient Methods for Inspection Sensitivity

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**Nondestructive evaluation (NDE) inspections play a vital role in the reliability of structural components. NDE inspecitons are simulated using a Probability-of-Detection curve (POD) that is obtained experimentally. In this paper a methodology using the expectation operator was developed to obtain the sensitivity of the Probability-of-Failure (POF) with respect to parameters of the POD via Monte Carlo Sampling. The methodology scales to any number of inspections and can be integrated with existing Monte Carlo sampling methods. Sensitivities of the probability-of-failure with respect to the parameters of a POD curve were calculated for several numerical examples then compared against analytical and numerical finite difference solutions.**

## Nomenclature

| | |
|---|---|
| $f(\theta,x)$ | crack size probability distribution before inspection |
| $f^{(n)}(x)$ | crack size probability distribution after nth inspection |
| $POD(\theta,x)$ | Probability of detection curve |
| $CPOD(\theta,x)$ | complementary POD, equals 1 minus POD |
| X | crack size random variable |
| $\theta$ | parameter of POD curve |
| $c(\theta)$ | scaling parameter such that the integral of $f'(\theta,x)$ equals one |
| $P_{Det}$ | probability of detecting a crack |
| $P_{NoDet}$ | probability of not detecting a crack |
| $P^{(n)}{}_f$ | probability of failure after nth inspection |
| $dP^{(n)}{}_f/d\theta$ | probability of failure sensitivity with respect to $\theta$ after nth inspection |
| $E_{f^{(n)}}[\bullet]$ | expected value operator with respect to $f^{(n)}(x)$ |

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| | |
|---|---|
| $N^{(n)}$ | number of samples generated from $f^{(n)}(x)$. $N^{(n)} = N^{(n)}{}_{Det} + N^{(n)}{}_{NoDet}$ |
| $N^{(n)}{}_{Det}$ | number of samples detected by the nth inspection process and removed |
| $N^{(n)}{}_{NoDet}$ | number of samples not detected by the nth inspection process |

## I. Introduction

THE POD curve has been seen as the "formal measure of quantifying NDT reliability".[2] The procedures used to create a POD take on the following sequence of events: create flaw specimens, inspect specimens with an NDT method, obtain results, then plot the POD curve as a function of flaw size.[2] Probabilistic sensitivity analysis proves to be an essential part of engineering design. Kulkarni has presented a method that optimizes the cost-effectiveness of multiple inspections based on each inspection's POD curve. His damage tolerance approach for when inspections should occur is based on three factors: macro crack initiation, macro crack growth, and component inspection.[6] Research into on board sensors that can detect cracks continues. For instance, Shook et. al discussed the effect of recurring automated inspections on probability-of-failure estimates using sensor performance POD curves.[5] The objective of this paper is to present an MCS method of obtaining sensitivities based on POD parameters. Sensitivities are critical when evaluating the fatigue life of a particular component and allow the assessment of the crucial parameters, which affect the POF. By knowing which parameters of a POD most affect the POF, the design and inspection protocol can be optimally modified.

The methodology developed here is to explicitly differentiate the equations to determine the probability-of-failure with respect to the parameters of the POD curves then integrate these equations using the same Monte Carlo sampling method as used to determine the POF. As a result, accurate derivatives can be obtained without any additional calculations since the same samples used to compute the POF are used to compute the sensitivities.

## II. Methodology

Let $f(x)$ represent the crack size PDF before inspection. Note the initial samples will be available from $f(x)$. However, during the inspection process, the detected samples are filtered out. The samples after each inspection, $N^n$, simulate the samples that would be generated from the PDF $f^{(n)}(x)$, where n denotes the current inspection. Thus, it is beneficial to write the equations in terms of $f^{(n)}(x)$ and not $f(x)$, since the equations are developed using the undetected samples. For example, expected values are written as $E_{f^{(n)}}[\bullet] \approx \frac{1}{N^{(n)}}\sum_{i=1}^{N^{(n)}}(\bullet)_i$ and not $E_f[\bullet] \approx \frac{1}{N}\sum_{i=1}^{N}(\bullet)_i$. As shown below, $f^{(n)}(x)$ is a function of $\theta$; therefore, more formally, we write $f^{(n)}(\theta,x)$.

**A. Formulation**

The crack size PDF before inspection is $f^{(n-1)}(x)$. The crack size PDF after an inspection is $f^{(n)}(\theta,x) = c(\theta)\cdot CPOD(\theta,x) f^{(n-1)}(x)$, where $CPOD(\theta,x) = 1 - POD(\theta,x)$, $POD(\theta,x)$ is the probability-of-detection of the nth inspection, $\theta$ represents the parameters of the $POD(\theta,x)$, $c(\theta)$ denotes a scaling parameter such that the integral over the entire domain of $f^{(n)}(\theta,x)$ equals one, e.g., $c^{(n)}(\theta) = 1/\int_{-\infty}^{\infty} CPOD(\theta,x) f^{(n-1)}(x)dx$. Note, $c(\theta)$ depends on $\theta$ but not *X*.

The probability of detecting a crack from the nth inspection is

$$P^{(n)}{}_{Det} = \int POD(\theta,x) f^{(n-1)}(x)dx \approx \frac{N^{(n)}{}_{Det}}{N^{(n)}}$$

where $N^{(n)}{}_{Det}$ denotes the number of samples that were detected and $N^{(n)}$ denotes the total number of samples used for inspection *n*, and the probability of missing a crack is

$$P^{(n)}{}_{NoDet} = \int (1 - POD(\theta,x)) f^{(n-1)}(x)dx \approx \frac{N^{(n)}{}_{NoDet}}{N^{(n)}}$$

where $N^{(n)}{}_{NoDet}$ denotes the number of samples that were not detected.

The probability-of-failure after inspection can be determined as

$$P_f^{(n)}(\theta) = \int_{-\infty}^{\infty} I(x) f^{(n)}(\theta,x)dx = E_{f^{(n)}}[I(x)] \approx \frac{1}{N^{(n)}}\sum_{i=1}^{N^{(n)}} I(\mathbf{x}_i) \qquad (1)$$

where $I(\mathbf{x})$ denotes the indicator function which is one in the failure domain and zero otherwise, and $E_{f^{(n)}}[\bullet]$ represents the expected value operator with respect to $f^{(n)}(x)$.

The sensitivity of the probability-of-failure with respect to a parameter of the POD curve $\theta$, can be written

$$
\begin{aligned}
&\frac{\partial P_f^{(n)}}{\partial\theta} = \int I(x)c(\theta)\cdot CPOD(\theta,x)f^{(n-1)}(x)dx = \\
&\int I(x)c(\theta)\cdot CPOD(\theta,x)f^{(n-1)}(x)\left\{\frac{\partial c}{\partial\theta}\frac{1}{c}+\frac{\partial CPOD(\theta,x)}{\partial\theta}\frac{1}{CPOD(\theta,x)}\right\}dx = \\
&\int I(x)f^{(n)}(\theta,x)\left\{\frac{\partial c}{\partial\theta}\frac{1}{c}+\frac{\partial CPOD(\theta,x)}{\partial\theta}\frac{1}{CPOD(\theta,x)}\right\}dx
\end{aligned} \tag{2}
$$

Using the substitution $\Omega(\theta,x)=\frac{\partial POD(\theta,x)}{\partial\theta}\frac{1}{CPOD(\theta,x)}$ for brevity, the sensitivity can be written

$$
\frac{\partial P_f^{(n)}}{\partial\theta} = \int I(x)\left\{\frac{\partial c}{\partial\theta}\frac{1}{c}-\Omega(\theta,x)\right\}f^{(n)}(\theta,x)dx \tag{3}
$$

The term $\frac{\partial c}{\partial\theta}\frac{1}{c}$ can be determined as follows. Define $L$ as

$$
L=\int_{-\infty}^{\infty} CPOD(\theta,x)\cdot f^{(n-1)}dx
$$

then

$$
\begin{aligned}
&\frac{\partial c}{\partial\theta}=\frac{\partial L^{-1}}{\partial\theta}=\frac{\partial}{\partial\theta}\left(\int_{-\infty}^{\infty} CPOD(\theta,x)\cdot f^{(n-1)}(x)dx\right)^{-1} = \\
&-\int\frac{\partial CPOD(\theta,x)}{\partial\theta}f^{(n-1)}(\mathbf{x})dx\left(\int_{-\infty}^{\infty} CPOD(\theta,x)\cdot f^{(n-1)}(x)dx\right)^{-2} \\
&=c^2\int\frac{\partial POD(\theta,x)}{\partial\theta}f^{(n-1)}(x)dx
\end{aligned} \tag{4}
$$

Therefore,

$$
\begin{aligned}
&\frac{\partial c}{\partial\theta}\frac{1}{c}=c\int\frac{\partial POD(\theta,x)}{\partial\theta}f^{(n-1)}(x)dx = \\
&\int\Omega(\theta,x)\cdot c\cdot CPOD(\theta,x)f^{(n-1)}(x)dx = \\
&\int\Omega(\theta,x)f^{(n)}(\theta,x)dx = \\
&E_{f^{(n)}}[\Omega(\theta,x)]
\end{aligned} \tag{5}
$$

Note that $\frac{\partial c}{\partial\theta}\frac{1}{c}$ is a function of $\theta$ but not *X*, therefore, this term can be extricated from the integral with respect to X and Eq. (3) becomes

$$\frac{\partial P_f^{(n)}}{\partial \theta} = \frac{\partial c}{\partial \theta}\frac{1}{c}\int I(x) f^{(n)}(\theta,x)dx - \int I(x)\Omega(\theta,x) f^{(n)}(\theta,x)dx \tag{6}$$

Substituting $P_f^n = \int_{-\infty}^{\infty} I(x) f^n(x)dx$ and the corresponding term for $\frac{\partial c}{\partial \theta}\frac{1}{c}$ from Eq. (5) yields

$$\begin{aligned}&\frac{\partial P_f^n}{\partial \theta} = P_f \int \Omega(\theta,x) f^{(n)}(\theta,\mathbf{x})dx - \int I(x)\Omega(\theta,x) f^{(n)}(\theta,x)dx = \\ &P_f E_{f^{(n)}}[\Omega(\theta,x)] - E_{f^{(n)}}[I(x)\Omega(\theta,x)]\end{aligned} \tag{7}$$

### B. Numerical implementation

The probability-of-failure and its sensitivity with respect to $\theta$ are determined by several integrals, see Eqs. (1,7). The integrals can be estimated by approximating the expected value operator using sampling methods, e.g., Monte Carlo, Latin hypercube, importance sampling, etc. Note, that the expectations are with respect to the *after* inspection pdf.

The necessary integrals can be approximated using sampling as

$$P_f \approx \frac{1}{N^{(n)}}\sum_{i=1}^{N^{(n)}} I(\mathbf{x}_i) \tag{8}$$

$$\frac{\partial c}{\partial \theta}\frac{1}{c} = E_{f'}[\Omega(\theta,\mathbf{x}_i)] \approx \frac{1}{N^{(n)}}\sum_{i=1}^{N^{(n)}} \Omega(\theta,\mathbf{x}_i) \tag{9}$$

$$E_{f^{(n)}}[I(\mathbf{x})\Omega(\theta,\mathbf{x})] \approx \frac{1}{N^{(n)}}\sum_{i=1}^{N^{(n)}} I(\mathbf{x}_i)\Omega(\theta,\mathbf{x}_i) \tag{10}$$

### Calculation procedure

Accumulate the following sums

$$A = \sum_{i=1}^{N^{(n)}} I(\mathbf{x}_i) \tag{11}$$

$$B = \sum_{i=1}^{N^{(n)}} \Omega(\theta, \mathbf{x}_i) \quad \textbf{(12)}$$

$$C = \sum_{i=1}^{N^{(n)}} I(\mathbf{x}_i) \Omega(\theta, \mathbf{x}_i) \quad \textbf{(13)}$$

Then,

$$P_f^{(n)} \approx A / N^{(n)} \quad \textbf{(14)}$$

$$\frac{\partial P_f^n}{\partial \theta} \approx P_f^{(n)} B / N^{(n)} - C / N^{(n)} \quad \textbf{(15)}$$

## Numerical Examples

A simple example problem is solved for which an exact analytical solution can be determined. The problem is solved with a linear POD curve and a lognormal POD curve. The problem consists of two inspections with only one random variable, the crack size and without any crack growth.

### Linear POD Numerical Example

The probability-of-failure and its sensitivities with respect to the parameter of a linear POD curve, $\theta_1$ and $\theta_2$, were computed for two inspections using Monte Carlo sampling. An initial sample size of three million samples was used for the first inspection. Detected samples were then removed (no repair) and the samples not detected were then used in the second inspection. To verify the equations, the Monte Carlo sampling results were compared to analytical and finite difference results. The results for the first inspection are shown in Table 1 and the second inspection can be seen in Table 2. In the linear case, the sensitivities for both inspections show that increasing the slope of the POD curve will result in a decrease in the POF.

### 1. First Inspection

The PDF before the first inspection was defined by

$$f(x)=\begin{cases} 208.3x & 0<x\le 0.06 \\ 20-125x & 0.06<x\le 0.16 \\ 0 & x>0.16 \end{cases}$$

and can be seen in Figure 1. The POD curve used for the first inspection was chosen as

$$POD_1(x)=\begin{cases} 0 & x\le 0 \\ 8x & 0<x\le 1/8 \\ 1 & x>1/8 \end{cases}$$

and can be seen in Figure 2, i.e. $\theta_1=8$.

The posterior PDF can be written

$$f^{(1)}(x)=\begin{cases} 0 & x\le 0 \\ 2.3785\cdot(1-8x)\cdot(208.3x) & 0<x\le 0.06 \\ 2.3785\cdot(1-8x)\cdot(20-125x) & 0.06<x\le 1/8 \\ 0 & x>1/8 \end{cases}$$

where 2.3785 is the scaling parameter, *c,* for the first inspection. The failure criterion was X ≥ 0.1. The probability-of-failure $P^{(1)}{}_f$ after the first inspection can then be calculated by

$$P_f^{(n)}(\theta)=\int_{0.1}^{\infty} I(x)f^{(n)}(\theta,x)dx \approx \frac{1}{N^{(n)}}\sum_{i=1}^{N^{(n)}} I(\mathbf{x}_i)$$

### 2. Second Inspection

A linear POD with a slope of $\theta_1=9$ was used for the second inspection process.

$$POD_2(x)=\begin{cases} 0 & x\le 0 \\ 9x & 0<x\le 1/9 \\ 1 & x>1/9 \end{cases}$$

The updated PDF for the second inspection given the scaling parameter is now

$$f^{(2)}(x)=\begin{cases} 0 & x\le 0 \\ 4.6071\cdot(1-8x)(1-9x)(208.3x) & 0<x\le 0.06 \\ 4.6071\cdot(1-8x)(1-9x)(20-125x) & 0.06<x\le 1/9 \\ 0 & x>1/9 \end{cases}$$

The scaling parameter $c_2$ is now 4.6071. Figure 3 shows the PDF pre-inspection and post-inspection for both cases.

The results for the POF and its sensitivity with respect to the slope of the POD curves are shown in Table 1.

## B. Lognormal POD Numerical Example

A second example was implemented with lognormal PODs, shown in Figure 4. The POD curves are defined by the equation $POD(x) = \frac{1}{\sqrt{2\pi}\varsigma x} Exp[-\frac{(\ln(x)-\lambda)^2}{2\varsigma^2}]$ where $\mu$ and $\sigma$ are the mean and standard deviation of X and $\lambda$ and $\varsigma$ are the mean and standard deviation of the log(X). The same piecewise PDF from the previous example was used as the initial crack size. The lognormal POD parameters for the inspections were $\mu_1 = .06$, $\sigma_1 = .02$ and $\mu_2 = .03$, $\sigma_2 = .02$. Monte Carlo sampling and analytical results for both inspections are shown in Table 3 and 4. A history of the PDF after each inspection can be seen in Figure 5.

**C. Fatigue Example**

An example problem considering fatigue crack growth was computed using the Paris Law (Stage II) crack growth model of the form $da/dN = C(\Delta K)^n$. Two linear POD, $\theta_1=\theta_2$=1636.72, inspections were performed for this example. The initial crack size in inches was modeled as a random variable with a lognormal distribution L[5.951E-4,3.344E-4]. Random variables C and n were obtained experimentally with distributions and parameters of N[log (-10.09), log (0.1570)], N[3.813,0.1456]. The correlation between C and n was $\rho_{Cn}$ = -.9751. The fracture toughness was 50 ksi-in$^{1/2}$ and constant amplitude loading with an R ratio of zero and a maximum load of 70ksi was applied. The critical crack size was 0.130 inches. With a critical crack size known, an MCS code was used to calculate the average number of cycles to failure for 1E5 samples, which was 70,000. It was arbitrarily chosen that the inspection would occur at ten and forty percent of the $N_f$ , 7,000 and 28,000.

For one run using MCS, crack growth was generated to 7,000 cycles. If a crack was detected, future crack growth was aborted and a new MC sample generated and analyzed. If not detected, the sample was used to calculate the POF and the sensitivities of the POF with respect to the POD parameters. The undetected crack was allowed to continue to grow and was then reevaluated at the second inspection of 28,000 cycles. The inspection process was repeated for the inspection two.

Tables 5 and 6 contain the results from the fatigue example. As seen in the previous case of the linear POD, the POF decreases as the slope increases. Updated PDFs after each inspection are shown in Figure 6 and 7. In both cases, the PDF shifts to the left after each inspection, which graphically displays a decrease in failure after each inspection.

## III. Conclusion

A methodology was presented such that the sensitivity of the POF with respect to the parameters of the POD curve of the inspection process can be obtained with high accuracy for negligible cost. The methodology is formulated in terms of expected value operators such that traditional sampling methods can be used, or, more likely, the additional equations needed to compute the sensitivities can be integrated with existing damage tolerance analysis codes. The methodology was demonstrated using three example problems and the sensitivities compared to analytical or numerical derivatives. The agreement in all cases was very close.

## Appendix

**Table 1. First Inspection Sensitivity for Linear POD**

| | Monte Carlo | Analytical | Finite Difference (2% forward difference) |
|---|---|---|---|
| $P_{f_{insp}}^{(1)}$ | 0.038495 | 0.038402 | - |
| $\partial P_{f_{insp}}^{(1)} / \partial \theta_1$ | -0.033842 | -0.033589 | -0.032794 |

Failure Criterion: P[x ≥ 0.1 in.] *Samples: 3E6

**Table 2. Second Inspection Sensitivity for Linear POD**

| | Monte Carlo | Analytical | Finite Difference (2% forward difference) |
|---|---|---|---|
| $P_{f_{insp}}^{(2)}$ | 0.003032 | 0.003086 | - |
| $\partial P_{f_{insp}}^{(2)} / \partial \theta_2$ | -0.005597 | -0.005422 | -0.005113 |

Failure Criterion: P[x ≥ 0.1 in.] *Samples: 1,260,826

**Table 3. First Inspection Sensitivities for Lognormal POD** $\mu_1 = .06$ , $\sigma_1 = .02$

| | **Monte Carlo** | **Analytical** | **MC/Analytical** |
|---|---|---|---|
| $P_f^{(1)}$ | 0.009358 | 0.009265 | 1.010 |
| $\partial P_f^{(1)} / \partial \lambda_1$ | 0.05444 | 0.05358 | 1.016 |
| $\partial P_f^{(1)} / \partial \varsigma_1$ | 0.1384 | 0.1350 | 1.025 |
| $\partial P_f^{(1)} / \partial \mu_1$ | 0.2875 | 0.2893 | 0.9938 |
| $\partial P_f^{(1)} / \partial \sigma_1$ | 1.860 | 1.811 | 1.027 |

Failure Criterion: P[x ≥ 0.1 in.] *Samples: 1E6

**Table 4. Second Inspection Sensitivities for Lognormal POD** $\mu_2 = .03$ **,** $\sigma_2 = .02$

| | **Monte Carlo** | **Analytical** | **MC/Analytical** |
|---|---|---|---|
| $P_f^{(2)}$ | 0.0002672 | 0.0002628 | 1.017 |
| $\partial P_f^{(2)} / \partial \lambda_1$ | 0.0008293 | 0.0008080 | 1.026 |
| $\partial P_f^{(2)} / \partial \varsigma_1$ | 0.002732 | 0.002650 | 1.031 |
| $\partial P_f^{(2)} / \partial \mu_1$ | -0.01007 | -0.009603 | 1.049 |
| $\partial P_f^{(2)} / \partial \sigma_1$ | 0.05656 | 0.05480 | 1.032 |

Failure Criterion: P[x ≥ 0.1 in.] *Samples: 375,517

**Table 5. First Inspection Sensitivities for Fatigue Example**

| | Monte Carlo | Finite Difference (.2% forward difference) |
|---|---|---|
| $P_{f_{insp}}^{(1)}$ | 3.109E-5 | - |
| $\partial P_{f_{insp}}^{(1)} / \partial \theta_1$ | -3.141E-6 | -3.924E-6 |

Failure Criterion: P[x ≥ 0.000606 in.] *Samples: 1E8

**Table 6. Second Inspection Sensitivities for Fatigue Example**

| | Monte Carlo | Finite Difference (.2% forward difference) |
|---|---|---|
| $P_{f_{insp}}^{(2)}$ | .4595 | - |
| $\partial P_{f_{insp}}^{(2)} / \partial \theta_2$ | -0.03283 | -0.03093 |

Failure Criterion: P[x ≥ 0.000606 in.] *Samples: 4,439,905

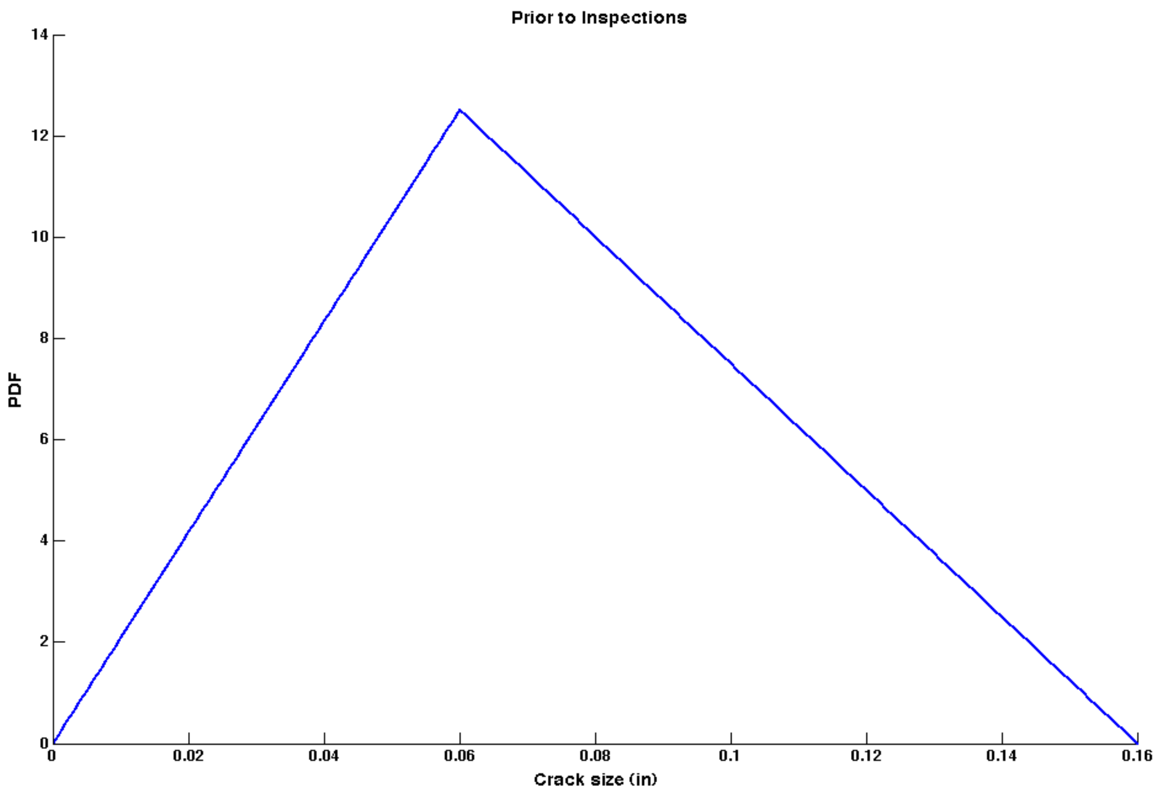


**Figure 1. Crack Size PDF Prior to Inspection**

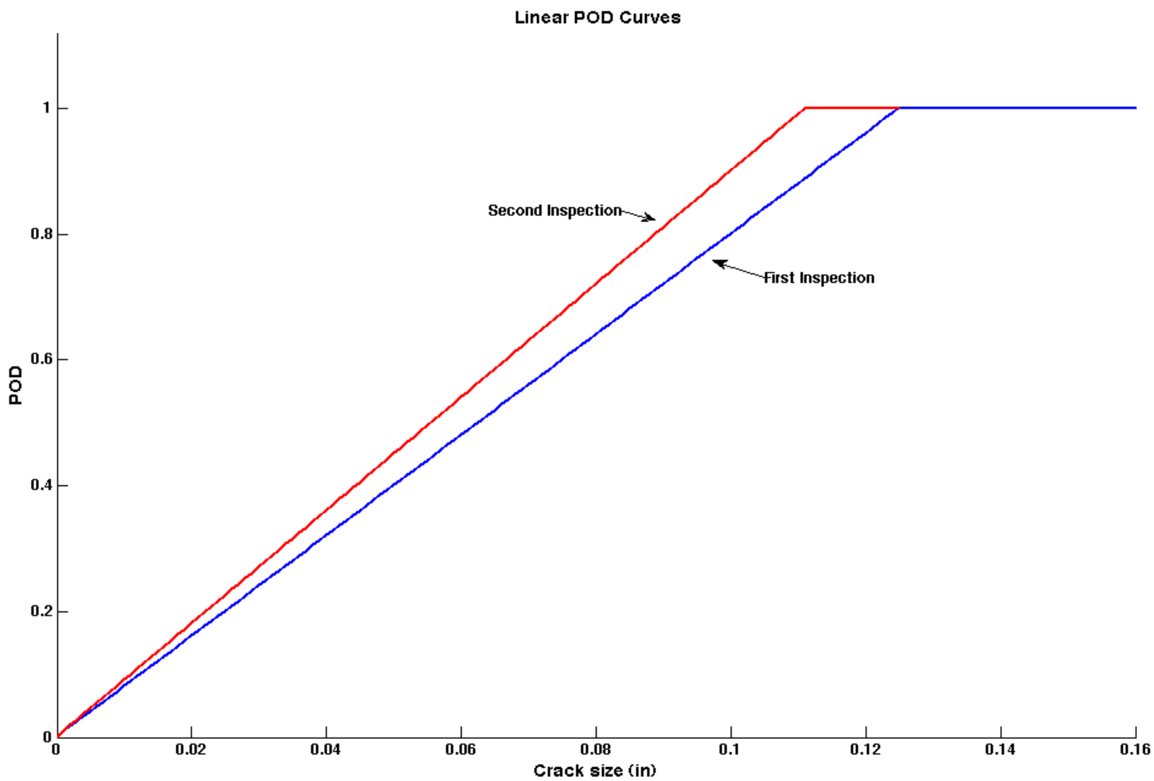


**Figure 2. Linear Probability-of-Detection Curves, $Slope_1$=8, $Slope_2$=9**

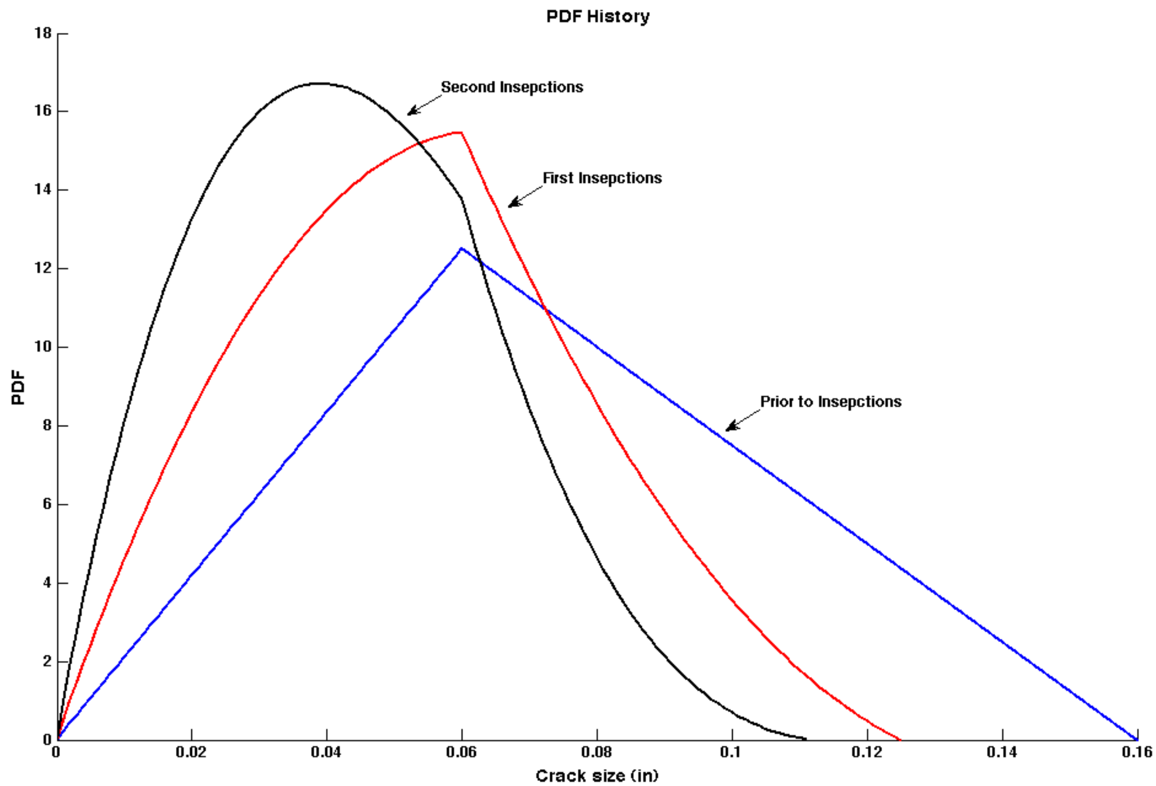


**Figure 3. Updated PDFs After Linear POD Inspections**

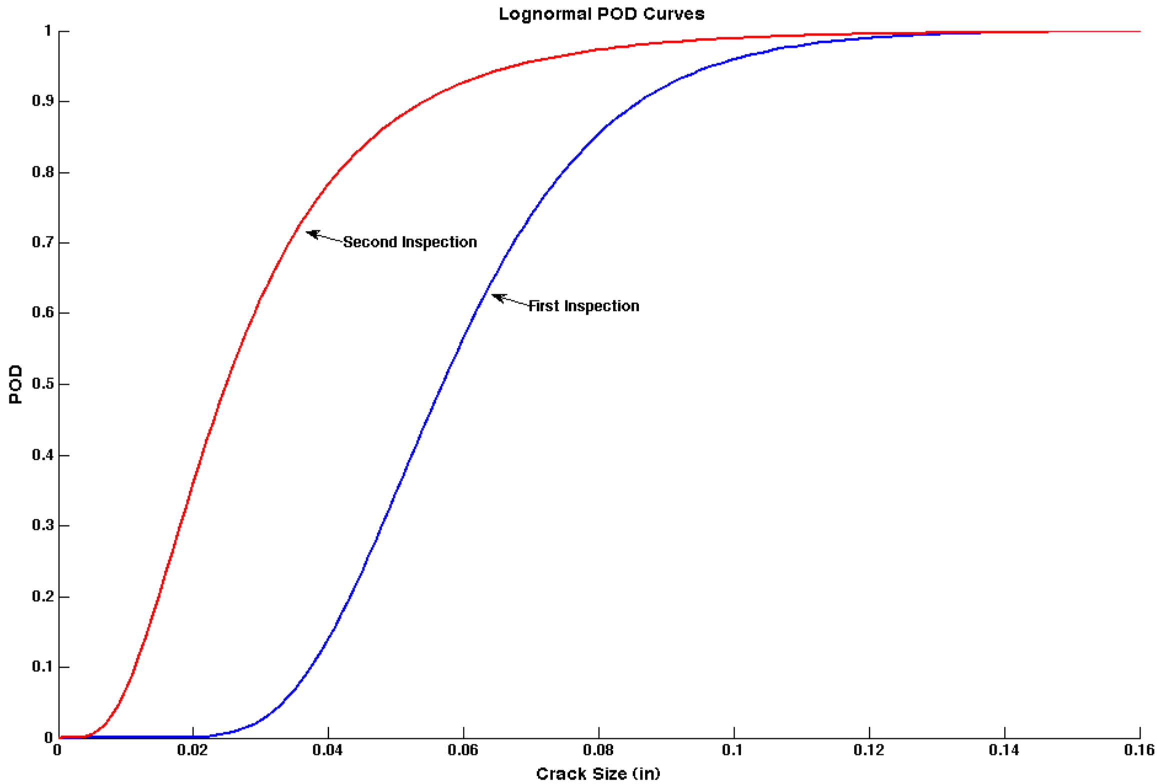


**Figure 4. Lognormal POD Curves,** $\mu_1 = .06,\ \sigma_1 = .02, \mu_2 = .03, \sigma_2 = .02$

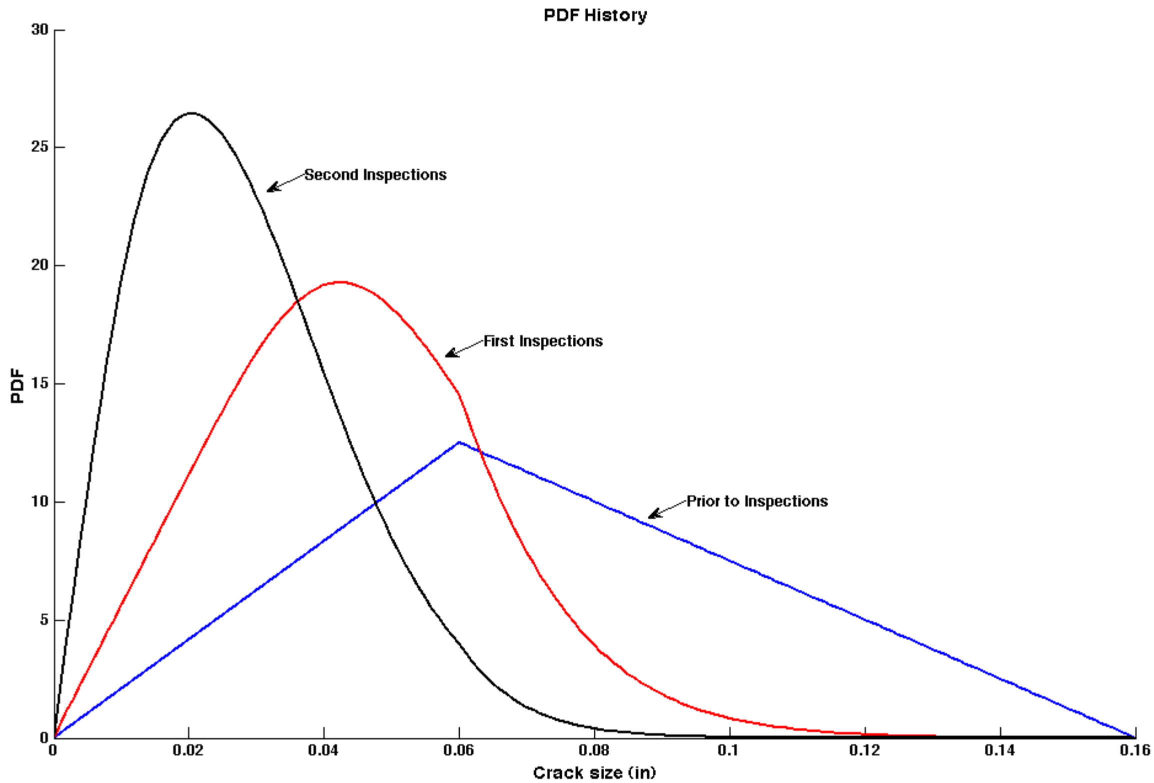


**Figure 5. Updated PDFs After Lognormal Inspections**

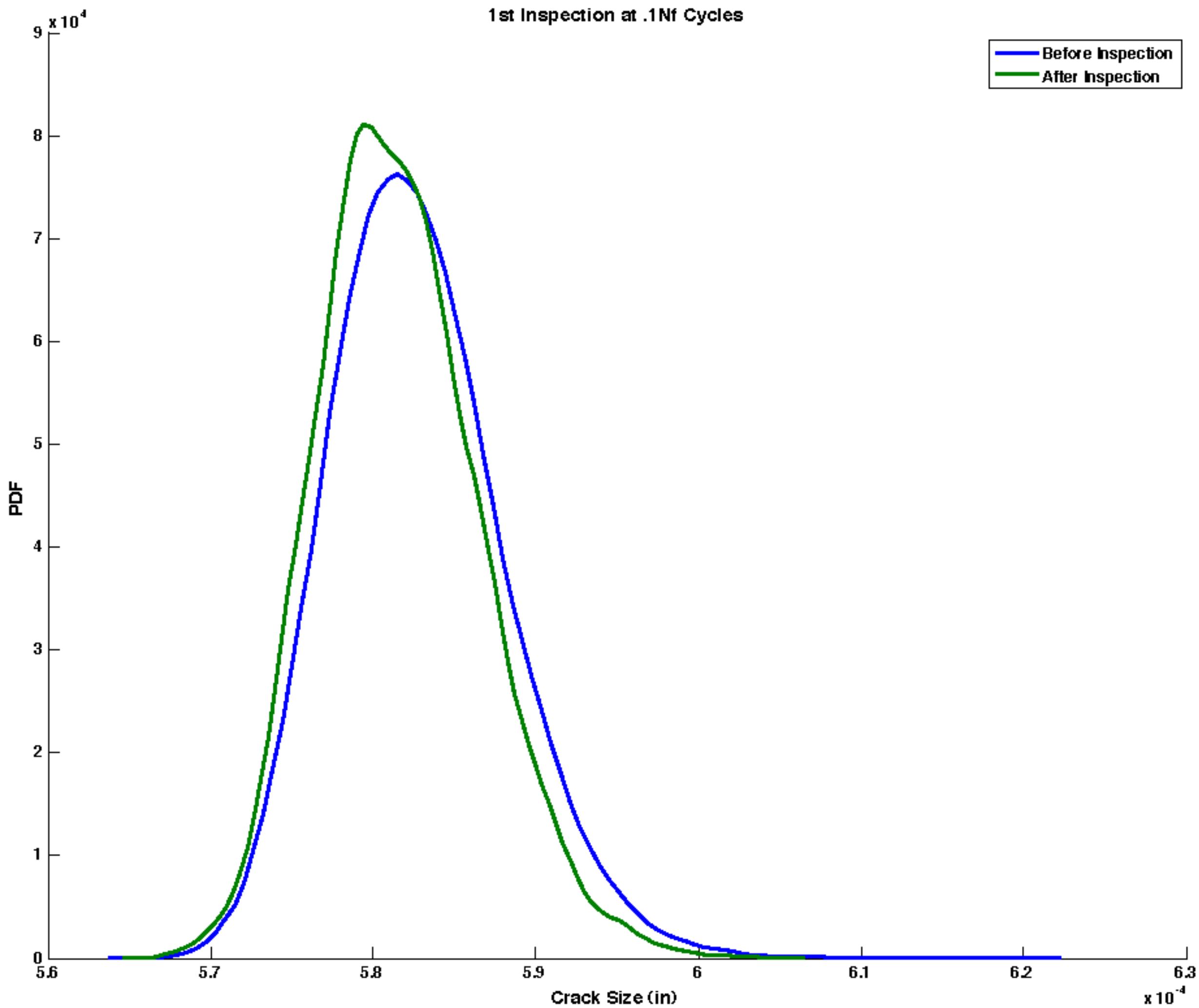


**Figure 6. Updated PDFs at First Inspection**

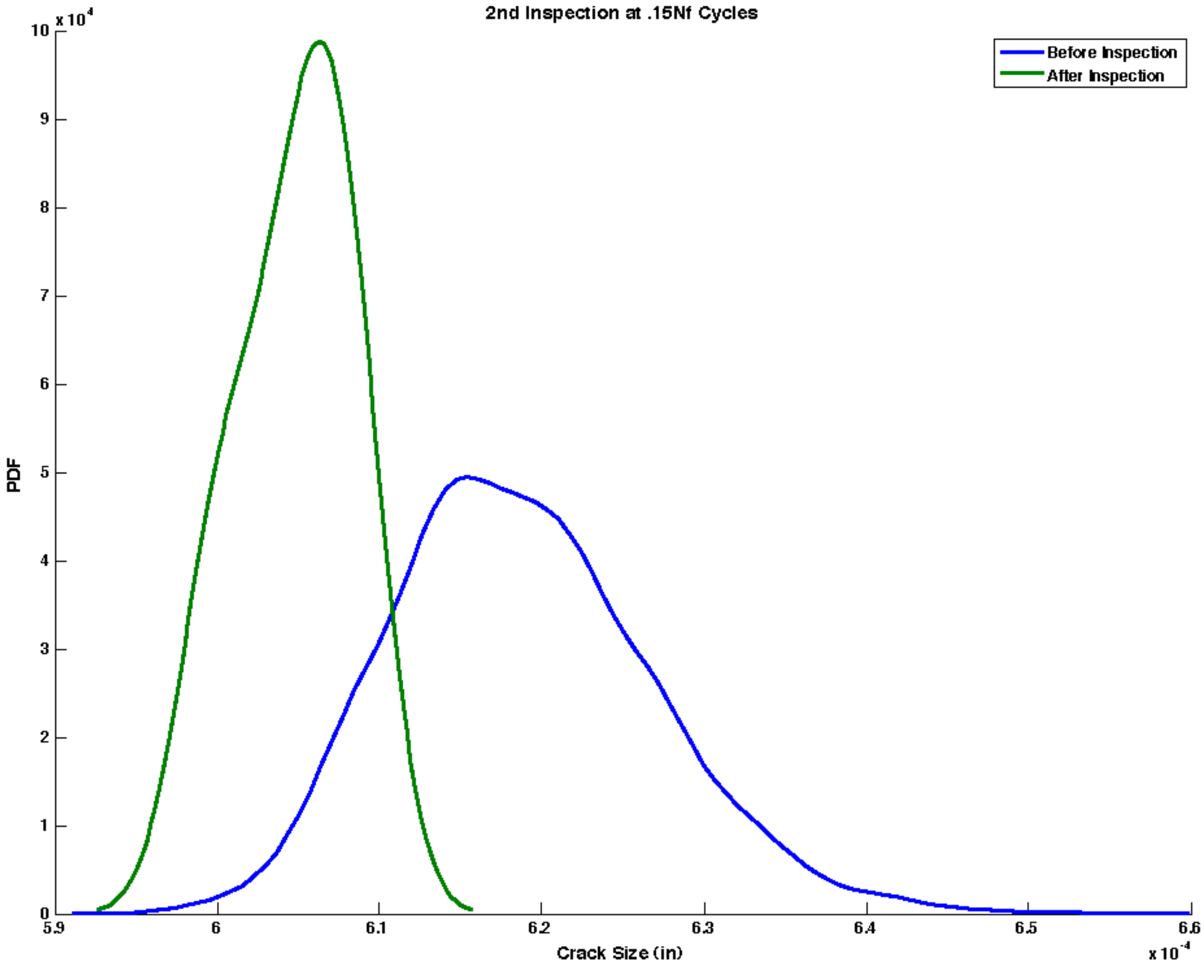


**Figure 7. Updated PDFs at Second Inspection**

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